UWThPh-2000-32

The seesaw mechanism at arbitrary order: disentangling the small scale from the large scale

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5 October 2000

Abstract

We develop a recipe which allows one to recursively and uniquely decouple the large scale from the small scale in mass matrices of the seesaw type, up to any order in the inverse of the large scale. Our method allows one to calculate the mass matrix of the light neutrinos with arbitrary precision. The same method can be applied in the case of quark mass matrices in an extension of the Standard Model with vector-like quarks which have mass terms at a scale much higher than the electroweak scale.

1 Introduction

Neutrino physics might prove to be the first window to physics beyond the Standard Model. The reason for this is the evidence for neutrino oscillations found, in particular, in experiments measuring the atmospheric neutrino flux [1]. Most probably, physics beyond the Standard Model is associated with a new mass or energy scale above the electroweak scale. This idea has been exploited in the seesaw mechanism to explain the smallness of the neutrino masses relative to the charged-lepton masses [2]. Inspired by the indication for non-zero neutrino masses from neutrino oscillations, model building for incorporating neutrino masses and lepton mixing and for explaining features of the

neutrino mass spectrum and of the mixing matrix is in full swing. In this context, the seesaw mechanism plays an important role [3].

Usually, the lowest order in the inverse heavy seesaw scale is sufficient for the purpose of describing the light-neutrino mass matrix. However, one can envisage situations which warrant going beyond the lowest order, like

- the need to know the neutrino masses to a better precision;
- a degeneracy of the mass spectrum at lowest order, which is lifted at higher orders;
- the presence of massless neutrinos at lowest order, which become massive at higher orders;
- a lepton mixing matrix with some features at lowest order, which disappear at higher orders [4];
- light mass terms which, though much smaller than the heavy ones, are not all of the same order of magnitude; or, inversely, heavy masses with different orders of magnitude [5]; or the same situation for both types of mass terms.

In this letter we propose a simple, minimal procedure which allows one to decouple the small seesaw scale from the large one order by order in the inverse large scale. The procedure is defined by a particular *Ansatz* for the unitary matrix W which performs this decoupling, together with a series expansion for W in the inverse large scale. This procedure enables, in principle, the calculation of W, of the mass matrix of the light neutrinos, and of the mass matrix of the heavy neutrinos, up to any arbitrarily large order in the inverse heavy mass scale.

Seesaw-type mass matrices, with two different mass scales, are not confined to neutrino physics. In this letter we also show how one may recursively decouple the light from the heavy quarks when one adds vector-like quarks to the Standard Model and one assumes the gauge-invariant mass terms to be very large.

In Section 2 we develop the decoupling procedure in the case of the usual seesaw mechanism for neutrinos, whereas in Section 3 we consider the extension of the Standard Model with vector-like quarks. In Section 4 we present our conclusions.

2 Neutrinos

Let us consider a general model in which, to the n_L generations of leptons of the Standard Model, one adds n_R right-handed (singlet) neutrinos ν_R [6]. One may take the charge conjugates of the latter fields, $\nu_R^c \equiv C\overline{\nu_R}^T$, which are left-handed, and put them together with the doublet left-handed neutrinos. (The matrix C is the Dirac–Pauli charge-conjugation matrix.) The most general mass terms for all the neutrinos fields are then

$$\mathcal{L}_{\mathrm{mass}} = \frac{1}{2} \left(\begin{array}{cc} \nu_L^T, & (\nu_R^c)^T \end{array} \right) \left(\begin{array}{cc} M_L & M_D^T \\ M_D & M_R \end{array} \right) C^{-1} \left(\begin{array}{c} \nu_L \\ \nu_R^c \end{array} \right) + \mathrm{H.c.} \tag{1}$$

The $n_L \times n_L$ matrix M_L and the $n_R \times n_R$ matrix M_R are symmetric; the matrix M_D is $n_R \times n_L$ and arbitrary.

The mass terms in M_D are $|\Delta I| = 1/2$, where I denotes the weak isospin, and originate, usually, in the Yukawa couplings of the leptons to scalar doublets which acquire a vacuum expectation value (VEV). The mass terms in M_L are $|\Delta I| = 1$ and are present if there exists in the theory either a fundamental or an effective Higgs triplet with a VEV. The mass terms in M_R are gauge-invariant and we shall assume them to be much larger than the ones in either M_D or M_L. We shall moreover assume M_R to be non-singular. The order of magnitude of the eigenvalues of $\sqrt{M_R^\dagger M_R}$ will be denoted by m_R. We shall assume that all matrix elements of M_L and of M_D are much smaller than m_R. Under these assumptions, there are in the theory n_R heavy neutrinos with mass of order m_R, and n_L light neutrinos with masses which are, in the case $M_L = 0$, suppressed by one or more inverse powers of m_R [2].[1] Note, however, that the notion "order of magnitude m_R" should not be taken too literally. To allow for a more general situation it should be understood in the sense that the eigenvalues of $\sqrt{M_R^\dagger M_R}$, though not necessarily all of the same order of magnitude, are all much larger than the matrix elements of M_L and M_D; the latter, too, may have orders of magnitude different among themselves.

It is our aim to decouple the heavy from the light neutrino fields, and to derive the effective mass matrices for each of them. This is done by performing a unitary transformation of the neutrino fields by means of an $(n_L + n_R) \times (n_L + n_R)$ unitary matrix W,

$$\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = W \begin{pmatrix} \nu_{\text{light}} \\ \nu_{\text{heavy}} \end{pmatrix}_L , \tag{2}$$

and by requiring from this unitary transformation that

$$W^T \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix} W = \begin{pmatrix} M_{\text{light}} & 0 \\ 0 & M_{\text{heavy}} \end{pmatrix} , \tag{3}$$

where M_{light} and M_{heavy} are, respectively, $n_L \times n_L$ and $n_R \times n_R$ symmetric matrices.

In Eq. (3) we require that the transformation performed by the matrix W has the property of leading to a zero $n_L \times n_R$ submatrix of the mass matrix. On these general grounds, W must have $n_L n_R$ degrees of freedom, and any degrees of freedom in W beyond these ones are not needed and might even render its computation unnecessarily complicated. We shall therefore make the following *Ansatz*[2] for W:

$$W = \begin{pmatrix} \sqrt{1 - BB^\dagger} & B \\ -B^\dagger & \sqrt{1 - B^\dagger B} \end{pmatrix} , \tag{4}$$

where B is an $n_L \times n_R$ matrix which must be fixed as a function of M_L, M_D, and M_R. The square roots in Eq. (4) should be understood as power series, for instance,

$$\sqrt{1 - BB^\dagger} = 1 - \tfrac{1}{2} BB^\dagger - \tfrac{1}{8} BB^\dagger BB^\dagger - \tfrac{1}{16} BB^\dagger BB^\dagger BB^\dagger - \ldots \tag{5}$$

[1] A "singular seesaw mechanism" [7], where the matrix M_R is singular, is also possible. If $\sqrt{M_R^\dagger M_R}$ has n_0 eigenvalues 0, then we can define $n_L' = n_L + n_0$ and $n_R' = n_R - n_0$, and this singular seesaw case in contained mathematically in the usual seesaw case with n_L' left-handed doublets and n_R' right-handed singlets. This is most easily seen by going into a basis where M_R is diagonal, with the n_0 diagonal zero entries coming first.

[2] In Ref. [8] an equivalent *Ansatz* has been proposed; Ref. [9] uses the *Ansatz* of Eq. (4) in a more restrictive context.

The matrix W in Eq. (4) is unitary by construction. Indeed, Eq. (4) might be seen as a generalization, for matrices, of the usual form of a 2×2 orthogonal matrix

$$\left(\begin{array}{cc} \sqrt{1-\sin^2\theta} & \sin\theta \\ -\sin\theta & \sqrt{1-\sin^2\theta} \end{array} \right).$$

With the *Ansatz* for W in Eq. (4), the condition of the vanishing of the off-diagonal submatrices in Eq. (3) reads

$$B^T M_L \sqrt{1-BB^\dagger} + \sqrt{1-B^TB^*} M_D \sqrt{1-BB^\dagger} - B^T M_D^T B^\dagger - \sqrt{1-B^TB^*} M_R B^\dagger = 0. \quad (6)$$

This equation may be solved by assuming that B is a power series in $1/m_R$, namely,

$$\begin{aligned} B &= B_1 + B_2 + B_3 + B_4 + \ldots, & (7) \\ \sqrt{1-BB^\dagger} &= 1 - \tfrac{1}{2} B_1 B_1^\dagger - \tfrac{1}{2}\left(B_1 B_2^\dagger + B_2 B_1^\dagger\right) \\ &\quad - \tfrac{1}{2}\left(B_1 B_3^\dagger + B_2 B_2^\dagger + B_3 B_1^\dagger + \tfrac{1}{4} B_1 B_1^\dagger B_1 B_1^\dagger\right) - \ldots, & (8) \end{aligned}$$

where B_j is by definition proportional to $(m_R)^{-j}$. The recursive solubility of Eq. (6) can be seen in the following way. At order $(m_R)^0$ Eq. (6) simply reads $M_D - M_R B_1^\dagger = 0$, and this fixes B_1. We then note that the ℓ-th order of $\sqrt{1-BB^\dagger}$ is a function of the B_j with $j < \ell$, *cf.* Eq. (8). Following this observation, an inspection of the k-th order of Eq. (6) shows that, except for a term $-M_R B_{k+1}^\dagger$, all terms only involve the B_j with $j \leq k$. In other words, the k-th order terms of Eq. (6) lead to an expression for B_{k+1} in terms of the B_j with $j \leq k$. This proves the recursive calculability of B.

From Eq. (6) one thus obtains

$$\begin{aligned} M_R B_1^\dagger &= M_D, & (9) \\ M_R B_2^\dagger &= M_R^{-1*} M_D^* M_L, & (10) \\ M_R B_3^\dagger &= M_R^{-1*} M_R^{-1} M_D M_L^* M_L - M_R^{-1*} M_D^* M_D^T M_R^{-1} M_D - \tfrac{1}{2} M_D M_D^\dagger M_R^{-1*} M_R^{-1} M_D. & (11) \end{aligned}$$

The expressions for the B_k with $k \geq 4$ are rather long and complicated, but no fundamental problem arises in their recursive computation. The effective mass matrices for the light and heavy neutrinos are then

$$\begin{aligned} M_\mathrm{light} &= M_L - M_D^T M_R^{-1} M_D - \tfrac{1}{2}\left(M_D^T M_R^{-1} M_R^{-1*} M_D^* M_L + M_L M_D^\dagger M_R^{-1*} M_R^{-1} M_D\right) \\ &\quad + \tfrac{1}{2}\Big[M_D^T M_R^{-1}\left(M_D M_D^\dagger M_R^{-1*} + M_R^{-1*} M_D^* M_D^T\right) M_R^{-1} M_D \\ &\quad - M_D^T M_R^{-1} M_R^{-1*} M_R^{-1} M_D M_L^* M_L - M_L M_L^* M_D^T M_R^{-1} M_R^{-1*} M_R^{-1} M_D\Big] + \ldots, & (12) \\ M_\mathrm{heavy} &= M_R + \tfrac{1}{2}\left(M_D M_D^\dagger M_R^{-1*} + M_R^{-1*} M_D^* M_D^T\right) \\ &\quad + \tfrac{1}{2}\left(M_D M_L^* M_D^T M_R^{-1} M_R^{-1*} + M_R^{-1*} M_R^{-1} M_D M_L^* M_D^T\right) + \ldots, & (13) \end{aligned}$$

respectively. Equations (12) and (13) are correct up to order $(m_R)^{-3}$ and $(m_R)^{-2}$, respectively.

In the most common case of the seesaw mechanism, which is for instance obtained by extending the Standard Model with right-handed neutrino singlets but avoiding the presence of any Higgs triplet, one has a vanishing matrix M_L. Then the recursive decoupling mechanism becomes much simpler because

$$M_L = 0 \quad \Rightarrow \quad B_2 = B_4 = B_6 = \ldots = 0. \tag{14}$$

Let us sketch a proof of Eq. (14) by induction. By direct calculation we find that $B_2 = 0$ when $M_L = 0$, see Eq. (10). Now we assume that $B_2 = B_4 = \ldots = B_{2\ell} = 0$ ($\ell \geq 1$). In order to calculate $B_{2\ell+2}$ from Eq. (6), we need to know all the B_j with $j \leq 2\ell + 1$. Then, according to the induction assumption, only the B_j with odd index j contribute to $B_{2\ell+2}$. We now notice that $\sqrt{1 - BB^\dagger}$ only has even powers of $1/m_R$ up to order $2\ell + 1$. Then, a term-by-term inspection of Eq. (6) with $M_L = 0$ reveals that all terms are zero at order $2\ell + 1$ except for the term $-M_R B_{2\ell+2}^\dagger$. Therefore, $B_{2\ell+2} = 0$, and this proves the theorem in Eq. (14). As a consequence, we have

$$\begin{aligned} B &= B_1 + B_3 + \ldots, & (15) \\ \sqrt{1 - BB^\dagger} &= 1 - \tfrac{1}{2} B_1 B_1^\dagger - \tfrac{1}{2}\left(B_1 B_3^\dagger + B_3 B_1^\dagger + \tfrac{1}{4} B_1 B_1^\dagger B_1 B_1^\dagger\right) - \ldots, & (16) \end{aligned}$$

with

$$\begin{aligned} M_R B_1^\dagger &= M_D, & (17) \\ M_R B_3^\dagger &= -M_R^{-1*} M_D^* M_D^T M_R^{-1} M_D - \tfrac{1}{2} M_D M_D^\dagger M_R^{-1*} M_R^{-1} M_D, & (18) \end{aligned}$$

and, using similar considerations as in the above proof by induction, we also find that only odd powers of $1/m_R$ are present in M_{light} and M_{heavy}, in particular,

$$\begin{aligned} M_{\mathrm{light}} &= -M_D^T M_R^{-1} M_D \\ &\quad + \tfrac{1}{2} M_D^T M_R^{-1} \left(M_D M_D^\dagger M_R^{-1*} + M_R^{-1*} M_D^* M_D^T\right) M_R^{-1} M_D + \ldots, & (19) \\ M_{\mathrm{heavy}} &= M_R + \tfrac{1}{2}\left(M_D M_D^\dagger M_R^{-1*} + M_R^{-1*} M_D^* M_D^T\right) + \ldots & (20) \end{aligned}$$

It is possible to proceed and derive, recursively, expressions for B_5, B_7, and so on, and also better approximations for M_{light} and for M_{heavy}. However, that task is tedious and complicated, since the number of terms in the polynomials increases very rapidly.

The subcase of the case $M_L = 0$ where the leading term in M_{light} (19) is zero has been studied in Ref. [9]. In this subcase one does not need recursive expansions, as one is able to give closed forms for all the relevant matrices. By using Eq. (6) and the definition $Z = M_R^{-1} M_D$, one readily verifies that [9]

$$M_L = 0\,, \quad M_D^T M_R^{-1} M_D = 0 \quad \Rightarrow \quad B = Z^\dagger \left(1 + ZZ^\dagger\right)^{-1/2} = \left(1 + Z^\dagger Z\right)^{-1/2} Z^\dagger\,. \tag{21}$$

Then, using Eq. (3), one finds after some algebra that

$$M_{\mathrm{light}} = 0 \quad \text{and} \quad M_{\mathrm{heavy}} = \left(1 + Z^* Z^T\right)^{1/2} M_R \left(1 + ZZ^\dagger\right)^{1/2}. \tag{22}$$

It is remarkable that the vanishing of the lowest order contribution to M_{light} in Eq. (19) entails the exact vanishing of the whole matrix.[3]

[3]We thank A. Pilaftsis and A. Joshipura for calling our attention to this interesting subcase.

3 Vector-like quarks

Let us consider that to the n_g generations of u and d quarks of the Standard Model one adds n' vector-like singlet quarks of, say, charge $-1/3$. One then has n_g left-handed quarks d_L, which are components of doublets of SU(2), together with n' left-handed quarks D_L, which are singlets of SU(2); the $n_g + n'$ right-handed quarks d_R are all singlets [10]. The mass term for these down-type quarks reads

$$\begin{pmatrix} \bar{d}_L, & \bar{D}_L \end{pmatrix} \mathcal{M}\, d_R + \text{H.c.}, \text{ with } \mathcal{M} = \begin{pmatrix} m \\ M \end{pmatrix}. \tag{23}$$

The matrix m is $n_g \times (n_g + n')$ and gives the $|\Delta I| = 1/2$ mass terms; the matrix M is $n' \times (n_g + n')$ and contains the SU(2)-invariant mass terms. We shall assume that the matrix elements of m are of order λ, and that the n' eigenvalues of $\sqrt{MM^\dagger}$ are all non-zero and of order Λ, with $\lambda/\Lambda \equiv \epsilon \ll 1$.

The squared-mass matrix for the left-handed quarks d_L and D_L is

$$\mathcal{M}\mathcal{M}^\dagger = \begin{pmatrix} mm^\dagger & mM^\dagger \\ Mm^\dagger & MM^\dagger \end{pmatrix}. \tag{24}$$

After diagonalization, there will be n_g "light" quarks with masses of order λ, and n' "heavy" quarks with masses of order Λ. One would like to decouple the light quarks from the heavy ones and to find separate squared-mass matrices for each category. This we shall do by means of a unitary transformation,

$$U^\dagger \mathcal{M}\mathcal{M}^\dagger U = \begin{pmatrix} D_{\text{light}} & 0 \\ 0 & D_{\text{heavy}} \end{pmatrix}, \tag{25}$$

where U is $(n_g + n') \times (n_g + n')$ unitary, while D_{light} and D_{heavy} are, respectively, $n_g \times n_g$ and $n' \times n'$ Hermitian (and positive definite) matrices.

In Eq. (25) we are, as a matter of fact, transforming an $n_g \times n'$ submatrix of $\mathcal{M}\mathcal{M}^\dagger$ to zero by means of a unitary transformation. It is therefore convenient to choose for the unitary matrix U of that transformation an *Ansatz* which has exactly $n_g n'$ (complex) degrees of freedom. We therefore write

$$U = \begin{pmatrix} \sqrt{1 - FF^\dagger} & F \\ -F^\dagger & \sqrt{1 - F^\dagger F} \end{pmatrix}, \tag{26}$$

where F is an arbitrary $n_g \times n'$ matrix, to be fixed as a function of m and of M. The square roots in Eq. (26) should be understood as power series, just as in Eq. (5). It is then obvious that the matrix U in Eq. (26) is indeed unitary; it is the analogue of the matrix W in the previous section.

The condition which fixes F is the demand that there is a zero submatrix in Eq. (25), namely,

$$\sqrt{1 - FF^\dagger}\, mm^\dagger F + \sqrt{1 - FF^\dagger}\, mM^\dagger \sqrt{1 - F^\dagger F} - FMm^\dagger F - FMM^\dagger \sqrt{1 - F^\dagger F} = 0. \tag{27}$$

We solve this equation by writing F as a power series in ϵ,

$$F = F_1 + F_3 + F_5 + \ldots, \tag{28}$$

where F_j has matrix elements of order ϵ^j. Notice that only terms with odd j are necessary. Defining

$$S \equiv \left(M M^\dagger \right)^{-1} = S^\dagger, \tag{29}$$

it is easy to find

$$\begin{aligned} F_1 &= m M^\dagger S, & (30) \\ F_3 &= m \left(m^\dagger m M^\dagger S - M^\dagger S M m^\dagger m M^\dagger S - \tfrac{1}{2} M^\dagger S^2 M m^\dagger m M^\dagger \right) S. & (31) \end{aligned}$$

Unfortunately, the expressions for the F_j become quite complicated when $j \geq 5$, although they may in principle be derived recursively without any problem.

One is thus able to decouple the light from the heavy quarks. One finds their respective squared-mass matrices to be given by

$$\begin{aligned} D_{\rm light} &= m \left(1 - M^\dagger S M \right) m^\dagger \\ & \quad - \tfrac{1}{2} m \left[\left(1 - M^\dagger S M \right) m^\dagger m M^\dagger S^2 M + M^\dagger S^2 M m^\dagger m \left(1 - M^\dagger S M \right) \right] m^\dagger \\ & \quad + \ldots & (32) \\ D_{\rm heavy} &= M M^\dagger + \tfrac{1}{2} \left(S M m^\dagger m M^\dagger + M m^\dagger m M^\dagger S \right) + \ldots & (33) \end{aligned}$$

The expressions of $D_{\rm light}$ and $D_{\rm heavy}$ have been given to subleading order in ϵ^2. Note that only even powers of ϵ appear in the expansions of these squared-mass matrices.

It is interesting to write down the equations above in a specific simple weak basis. By performing a unitary mixing of the right-handed singlet quarks d_R and also of the left-handed singlet quarks D_L, one finds a weak basis in which

$$m = \left(\begin{array}{cc} G, & J \end{array} \right) \text{ and } M = \left(\begin{array}{cc} 0, & \hat{M} \end{array} \right), \tag{34}$$

where $\hat{M}$ is a diagonal and positive $n' \times n'$ matrix, while G and J are general $n_g \times n_g$ and $n_g \times n'$ matrices, respectively. In this weak basis one finds

$$\begin{aligned} D_{\rm light} &= G G^\dagger - \tfrac{1}{2} \left\{ G G^\dagger, J \hat{M}^{-2} J^\dagger \right\} + \ldots, & (35) \\ D_{\rm heavy} &= \hat{M}^2 + \tfrac{1}{2} \left(\hat{M}^{-1} J^\dagger J \hat{M} + \hat{M} J^\dagger J \hat{M}^{-1} \right) + \ldots, & (36) \end{aligned}$$

where $\{X, Y\} \equiv XY + YX$ is the anticommutator. In this basis it is also obvious that the $(n_g + n') \times (n_g + n')$ matrix $M^\dagger S M$, which appears in Eq. (32), is a projector, which projects into the space characterized by the last n' indices. Therefore, at lowest order, in the matrix $D_{\rm light}$ of Eq. (32) only the left $n_g \times n_g$ submatrix of m contributes. The degrees of freedom beyond the Standard Model, which are characterized in the weak basis of Eq. (34) by the last n' indices, do not contribute to the light-quark masses, as expected.

4 Conclusions

In this note we have discussed a method which allows to separate, order by order in the inverse seesaw scale, the small and large scales in seesaw-type mass matrices, and to obtain a recursive expansion for the light-neutrino and for the heavy-neutrino mass matrices. This has been achieved by the *Ansatz* of Eq. (4) for the matrix W, which approaches the unit matrix when the large scale tends to infinity. A similar method, with the matrix U of Eq. (26), can be used to decouple the light quarks from additional heavy quarks in extensions of the Standard Model with vector-like quarks. The lepton mixing matrix and the quark mixing matrix can then be calculated from the matrices M_{light} of Eq. (12) and D_{light} of Eq. (32), respectively. For this purpose, the lowest order of these matrices might be sufficient, yet in some instances it might be necessary to include further terms in order to obtain trustworthy representations of the mixing matrices. If one wants to go beyond lowest-order mixing, then in the neutrino case there are contributions from higher orders in M_{light} and in W; the same holds in the vector-like-quark case for D_{light} and U.

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